Quark Mass Corrections to the Bjorken and Gross–Llewellyn-Smith Sum Rules

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Abstract

Quark mass corrections to the spin partonic structure function $g_1(x, Q^2)$ and function $F_3(x, Q^2)$ are obtained at the order $O(\alpha_s)$ along with the coefficient functions $C^{(A)}$ and $C^{(V)}$ related to the Bjorken and Gross–Llewellyn-Smith sum rules. In the massless limit the difference between F_3 and g_1 is encountered to be $(\alpha_s/\pi)C_F(1-x)$. The results for the functions $C^{(A)}$ and $C^{(V)}$ at $m = 0$ agree with the previous MS-scheme calculations $C^{(A)} = C^{(V)} = 1 - (3\alpha_s/4\pi)C_F$.

Recently many theoretical efforts were spent to study radiative corrections to the polarized Bjorken sum rule (BSR) [1] in deep inelastic eN-scattering

$$\int_0^1 \left(g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)\right) \mathrm{d}x = \frac{1}{3}\left|\frac{g_A}{g_V}\right| C^{(A)}, \tag{1}$$

where g_A and g_V are the constants in nuetron weak decay and $C^{(A)}$ is the coefficient function of the axial current in the operator product expansion of two vector currents. Corrections to $C^{(A)}$ are known to coincide with those to another coefficient function $C^{(V)}$ which is relevant for the Gross–Llewellyn-Smith (GLS) sum rule [2] in neutrino-nucleon scattering

$$\int_0^1 \left(F_3^{\bar{\nu}p}(x,Q^2) + F_3^{\nu p}(x,Q^2)\right) \mathrm{d}x = 3C^{(V)}. \tag{2}$$

The leading corrections to BSR and GLS were first computed in Refs. [3, 4] and appeared to be $C^{(A)} = C^{(V)} = 1 - C_F(3\alpha_\mathrm{s}/4\pi)$. Next to leading order results for both sum rules can be found in Ref. [5]. Discrepancies between $C^{(A)}$ and $C^{(V)}$ arise only at order $O(\alpha_\mathrm{s}^3)$ [6] where 'light-by-light' disgrams appear. These calculations were performed in MS scheme with quark mass $m = 0$. From the other hand recent time mass dependent RG equations [7] and power corrections [8] attract a great attention. Thus it is of interest to get mass dependence of coefficient functions.

In the recent work [9] there were computed $O(\alpha_\mathrm{s})$ corrections to $C^{(A)}$ in on-shell scheme. It was noticed that there are contributions to the partonic structure function g_1 that survive when $m \to 0$. That is the diagram of the box type which is responsible for that. After momentum integration it develops an additional compensating factor $1/m^2$ which cancels with the mass in numerator resulting in finite terms. Thus the result for g_1 is different in dependence on whether mass $m = 0$ from the very beginning or it is kept till the end of calculations.

Below the partonic structure functions both g_1 and F_3 at order $O(\alpha_\mathrm{s})$ with $m \neq 0$ will be presented. Along this paper we use the on-shell renormalization scheme.

First we note that the contribution of virtual gluon can be expressed in terms of renormalized elastic formfactors of currents. Corrections to the vector (axial) current are usually written as ($q = p' - p$)

$$V_\mu = \bar{u}(p')\left\{\gamma_\mu\Big(1 + \frac{\alpha_\mathrm{s}}{\pi}C_F\mathcal{F}_1^V(q)\Big) + \frac{1}{4m}[\hat{q},\gamma_\mu]_- \frac{\alpha_\mathrm{s}}{\pi}C_F\mathcal{F}_2^V(q)\right\}u(p), \tag{3}$$

$$A_\mu = \bar{u}(p')\left\{\gamma_\mu\gamma_5\Big(1 + \frac{\alpha_\mathrm{s}}{\pi}C_F\mathcal{F}_1^A(q)\Big) + \frac{1}{4m}(p+p')_\mu\gamma_5 \frac{\alpha_\mathrm{s}}{\pi}C_F\mathcal{F}_2^A(q)\right\}u(p). \tag{4}$$

These formfactors were computed earlier in QED and electroweak theory (see e.g. [10] and references therein). We take them in the following form

$$
\begin{aligned}
\mathcal{F}_1^V = & -\left(1+\frac{1+\theta^2}{1-\theta^2}\log\theta\right)\log\frac{\mu}{m} - 1 - \frac{3\theta^2+2\theta+3}{4(1-\theta^2)}\log\theta \\
& + \frac{1+\theta^2}{1-\theta^2}\left(-\frac{1}{4}\log^2\theta + \frac{1}{2}\zeta(2) + \mathrm{Li}_2(-\theta) + \log\theta\log(1+\theta)\right), \qquad (5) \\
\mathcal{F}_2^V = & -\frac{\theta}{1-\theta^2}\log\theta, \qquad (6) \\
\mathcal{F}_1^A = & \mathcal{F}_1^V + \frac{\theta}{1-\theta^2}\log\theta, \qquad (7)
\end{aligned}
$$

with

$$
\theta = \frac{\sqrt{1+4r}-1}{\sqrt{1+4r}+1}, \qquad r = \frac{m^2}{Q^2}. \qquad (8)
$$

In the above formulae μ is a small 'gluon mass' ($\mu \ll t, m^2$) being infrared regulator and m is a quark mass. Formfactor $\mathcal{F}_2^A$ is irrelevant and hence is omitted. The vector current is normalized such that $\mathcal{F}_1^V(q=0)=1$. If such the condition is imposed then for the axial current one gets $\mathcal{F}_1^A(0) = 1-(\alpha_{\mathrm{s}}/2\pi)$. Formfactor $\mathcal{F}_2^V(0)=1/2$ is the well known anomalous magnetic moment. Let us emphasize that it is impossible to normalize both currents to unity in the presence of a nonvanishing mass. We shall return to this question later on.

Using definitions (3),(4) the virtual contributions can be cast into the form

$$
\begin{aligned}
g_1^{\mathrm{virtual}} = & \frac{1}{2}\frac{\alpha_{\mathrm{s}}}{4\pi}C_F\delta(1-x)\left\{8\mathcal{F}_1^V + 4\mathcal{F}_2^V\right\}, \qquad (9) \\
F_3^{\mathrm{virtual}} = & \frac{1}{2}\frac{\alpha_{\mathrm{s}}}{4\pi}C_F\delta(1-x)\left\{4\mathcal{F}_1^V + 4\mathcal{F}_1^A\right\}. \qquad (10)
\end{aligned}
$$

Next we turn to the real gluon contributions. Again one faces with IR divergencies which are due to soft or collinear gluon emission. As above they are regularized by letting a gluon have a small nonzero mass μ. Calculating strightforwardly the diagrams one obtains functions which have a nontrivial dependence on μ. It causes singularities like $1/(1-x)$ at the end point of x-integration (in presence of a small mass μ x is varying in the interval $0 < x < 1 - 2r\mu/m$). Of course after integration over variable x the μ dependence is cast into that as in formulae (9),(10) with the opposite sign so that μ cancels in the whole moments. To cancel out μ before integration we rewrite result using the well known 'plus distribution' [11], i.e. every function $f(x)$ being singular at $x=1$ is

replaced by

$$f(x) = f_+(x) + \delta(1-x) \int\limits_0^{1-2r\mu/m} f(z)\,\mathrm{d}z, \tag{11}$$

where the limit $\mu \to 0$ is implied. After some transformations the structure functions can be presented as

$$w^{\mathrm{real}}(x) = \frac{1}{2}\frac{\alpha_{\mathrm{s}}}{4\pi}C_F\left\{\delta(1-x)\mathcal{R} + 4(1+2r)\left(\frac{L}{1-x}\right)_+ - \frac{8}{(1-x)_+} + c_1 + c_2 L\right\}, \tag{12}$$

$$L = \frac{1}{\sqrt{1+4rx^2}}\log\frac{1+2rx+\sqrt{1+4rx^2}}{1+2rx-\sqrt{1+4rx^2}}. \tag{13}$$

Here $w(x)$ stands for either g_1 or F_3. All infrared divergencies now are absorbed in coefficients $\mathcal{R}$. By explicit calculation we have found for $\mathcal{R}, c_1$ and c_2

$$\mathcal{R}^{g_1} = -8\mathcal{F}_1^V - 4\mathcal{F}_2^V - 4 + 8\log r - 2\frac{5\theta^2+4\theta+5}{1-\theta^2}\log\theta \tag{14}$$

$$\mathcal{R}^{F_3} = -4\mathcal{F}_1^V - 4\mathcal{F}_1^A - 4 + 8\log r - 10\frac{1+\theta^2}{1-\theta^2}\log\theta \tag{15}$$

$$\begin{aligned} c_1^{g_1} = {} & \frac{1}{(1+4rx^2)^2(1-x+rx)^2}\Big[(1-x)(3+6x-8x^2) \\ & + 4rx(1-x)(2+15x-15x^2+4x^3) \\ & + 4r^2x^2(1+24x-29x^2+6x^3) + 8r^3x^4(5-x)\Big] \end{aligned} \tag{16}$$

$$\begin{aligned} c_1^{F_3} = {} & \frac{1}{(1+4rx^2)(1-x+rx)^2}\Big[(1-x)(7-9x-2x-4x^2) \\ & + 2rx(7-3x-4x^3) + 8r^2x^2\Big] \end{aligned} \tag{17}$$

$$\begin{aligned} c_2^{g_1} = {} & \frac{-2}{(1+4rx^2)^2}\Big[1+x+2r(2+x+9x^2-4x^3) \\ & + 4r^2x^2(8+5x-x^2) + 64r^3x^4\Big] \end{aligned} \tag{18}$$

$$c_2^{F_3} = \frac{-2}{1+4rx^2}\Big(1+x+4r(1+x)+16r^2x^2\Big) \tag{19}$$

From the formulae (9),(10) and (12)–(15) one can see that $\log\mu$'s cancel as well as $\mathcal{F}_{1,2}^{A,V}$ and we are left with IR safe expressions.

If $Q^2 \gg m^2$ then the formulae for g_1, F_3 greatly simplify. Adding the Born contribution and keeping only leading terms in m^2/Q^2 we arrive at

$$\begin{aligned} g_1(x) = {} & \frac{1}{2}\delta(1-x) + \frac{1}{2}\frac{\alpha_{\mathrm{s}}}{4\pi}C_F\Big\{2\Big[\frac{3}{2}\delta(1-x) + \frac{1+x^2}{(1-x)_+}\Big]\log\frac{Q^2}{m^2} - 5\delta(1-x) \\ & - \frac{7}{(1-x)_+} - 4\left(\frac{\log(x(1-x))}{1-x}\right)_+ + 2 + 8x + 2(1+x)\log(x(1-x))\Big\} \end{aligned} \tag{20}$$

$$F_3(x) = \frac{1}{2}\delta(1-x) + \frac{1}{2}\frac{\alpha_{\rm s}}{4\pi}C_F\Big\{2\Big[\frac{3}{2}\delta(1-x) + \frac{1+x^2}{(1-x)_+}\Big]\log\frac{Q^2}{m^2} - 5\delta(1-x)$$

$$- \frac{7}{(1-x)_+} - 4\left(\frac{\log(x(1-x))}{1-x}\right)_+ + 6 + 4x + 2(1+x)\log(x(1-x))\Big\} \quad (21)$$

$$(22)$$

As it was mentioned in the beginning of the paper corrections to the structure function F_3 evaluated in massless theory coincide with those to g_1. Here we see from (20),(21) that this is not true in massless limit of the massive formulae. The difference is totally due to coefficients c_1's. It is worth noting that the result obtained should not depend on an infrared regularization procedure provided quark mass m tends to zero after IR singularities are canceled out. Eqns. (20),(21) yield that g_1 develops an extraterm $-C_F(\alpha_{\rm s}/2\pi)(1-x)$. In fact this term defines a fermion helicity-flip probability $P_{+-}(x)$ and was studied in Ref. [12].

Using formulae (14)–(19) we obtain first moments of g_1 and F_3 with no approximation made

$$2\int_0^1 w(x)\,{\rm d}x = 1 + \frac{\alpha_{\rm s}}{4\pi}C_F\delta, \qquad w(x) = g_1(x), F_3(x). \quad (23)$$

Where the factors δ's read

$$\delta^{g_1} = -4 + \frac{1-16r}{2}I(r) - \frac{2-17r+16r^2}{2r(1-r)}\log r + \frac{1-5r-28r^2}{r\sqrt{1+4r}}\log\theta, \quad (24)$$

$$\delta^{F_3} = -\frac{5-4r}{1-r} - 8rI(r) + \frac{1+6r-16r^2+8r^3}{r(1-r)}\log r - \frac{1+10r+24r^2}{r\sqrt{1+4r}}\log\theta. \quad (25)$$

Function $I(r)$ can be expressed through Euler dilogarithm function ${\rm Li}_2$ and has the following representation

$$I(r) = \int_0^1 \frac{{\rm d}x}{\sqrt{1+4rx^2}}\log\frac{1+2rx+\sqrt{1+4rx^2}}{1+2rx-\sqrt{1+4rx^2}}$$

$$= \frac{1}{2\sqrt{r}}\{{\rm Li}_2(-t) - {\rm Li}_2(t) + {\rm Li}_2(-at) + {\rm Li}_2(\frac{a}{t}) - \frac{1}{2}{\rm Li}_2(a^2) + \frac{\pi^2}{4} + \frac{1}{2}\log^2 t\}, \quad (26)$$

$$a = (1-\sqrt{r})/(1+\sqrt{r}), \qquad t = \sqrt{1+4r} - 2\sqrt{r}. \quad (27)$$

In high Q^2 region it turns to be $I(r) = 2 - \log r + r(2/9 + (5/3)\log r) + O(r^2\log r)$.

Eqns. (23)–(25) together with (26) give the values of the first moments of the partonic structure functions in the whole region of $Q^2 > 0$. In deep inelastic case $(m^2/Q^2 \to 0)$ we

obtain $g_1^{(1)} = 1 - C_F(5\alpha_s/4\pi)$ while within the massless approach it is $1 - C_F(3\alpha_s/4\pi)$. This result was found earlier in Ref. [9].

Let us consider now the coefficient functions. The OPE says that

$$2g_1^{(1)}(Q^2) = C^{(A)}(Q^2)A_5^{(1)}, \tag{28}$$

$$2F_3^{(1)}(Q^2) = C^{(V)}(Q^2)A^{(1)}, \tag{29}$$

where A's are defined from operator matrix elements

$$\langle p, s|\bar{\psi}(q)\gamma_5\gamma_\mu\psi(q)|p', s\rangle = \bar{u}(p)\gamma_\mu\gamma_5 u(p')A_5^{(1)}(t), \tag{30}$$

$$\langle p|\bar{\psi}(q)\gamma_\mu\psi(q)|p'\rangle = \bar{u}(p)\gamma_\mu u(p')A^{(1)}(t), \tag{31}$$

$$t = (p' - p)^2, \tag{32}$$

with quark field ψ. Matrix elements (30),(31) must be taken at zero momentum transfer $t = 0$ in the on-shell scheme. For the vector current the identity $A^{(1)} \equiv 1 + (\alpha_s/\pi)\mathcal{F}_1^V(t = 0)$ follows and it is equal to unity because of the current normalization while for the axial we have $A_5^{(1)}(t) \equiv 1 + (\alpha_s/\pi)(\mathcal{F}_1^A(t) - \mathcal{F}_1^V(t))$. The structure in the parentheses looks like

$$\mathcal{F}_1^A(t) - \mathcal{F}_1^V(t) = \frac{\theta}{1-\theta^2}\log\theta, \tag{33}$$

with θ defined as in (8) and $r = m^2/t$. For large t (33) vanishes as it should be due to chiral invariance. This situation is realized when m is identically equal to zero, when the limit $t \to 0$ corresponding to the forward matrix element could be easily taken. However, if one take this limit *before* setting $m = 0$ the mass terms come into the game and (33) becomes $-1/2$. One can check this using (5)–(8). As a result $C^{(A)}$ differs form the $g_1^{(1)}$ at $m = 0$ by a finite term

$$C^{(A)} = \left(1 - \frac{5\alpha_s}{4\pi}C_F\right)\left(1 - \frac{\alpha_s}{2\pi}C_F\right)^{-1} = 1 - \frac{3\alpha_s}{4\pi}C_F + O(\alpha_s^2). \tag{34}$$

Let us summarize now the results. Coefficient functions C's with a nonvanishing fermion mass look like

$$C^{(A)} = 1 + \frac{\alpha_s}{4\pi}C_F(\delta^{g_1} + 2), \tag{35}$$

$$C^{(V)} = 1 + \frac{\alpha_s}{4\pi}C_F\delta^{F_3}, \tag{36}$$

where functions δ are given by (24),(25). In Fig.1 there are the plots of the corrections to coefficient functions $C^{(A)}, C^{(V)}$ versus m^2/Q^2. In the deep inelastic limit both corrections coincide with each other in agreement with the values quoted in literature.

Up to the terms $O(m^2/Q^2)^2$ the Eqns. (35),(36) read

$$C^{(A)}(Q^2) = 1 - \frac{\alpha_s}{4\pi} C_F \left[3 + \frac{m^2}{Q^2} \Big(\frac{11}{3} \log \frac{m^2}{Q^2} - \frac{10}{9} \Big) \right], \tag{37}$$

$$C^{(V)}(Q^2) = 1 - \frac{\alpha_s}{4\pi} C_F \left[3 + \frac{m^2}{Q^2} \Big(3 \log \frac{m^2}{Q^2} + 4 \Big) \right]. \tag{38}$$

The discrepancy between $C^{(A)}$ and $C^{(V)}$ manifests a violation of the Crewther relation [13] by mass corrections.

The magnitude of the strange quark mass correction is about 0.12 of the massless one-loop result at $Q^2 = 2\,\mathrm{GeV}^2$. For the light quarks, the mass contribution is negligible if one uses the current quark mass of order of few MeV. If, however, one takes into account that such a scale should be non-observable and substitute instead the scale of order of a pion mass [14], the result is still about 0.1.

The calculated corrections are in fact the first example of the NLO mass dependence in QCD. Generally speaking this would require to calculate the 2-loop mass-dependent anomalous dimension and one-loop coefficient function. However in the case at hand the anomalous dimension is zero and the calculated contribution provides the final result.

After this work was completed, the paper [15] appeared, where the coefficient functions for heavy quarks are investigated in the limit $Q^2 \geq m^2$.

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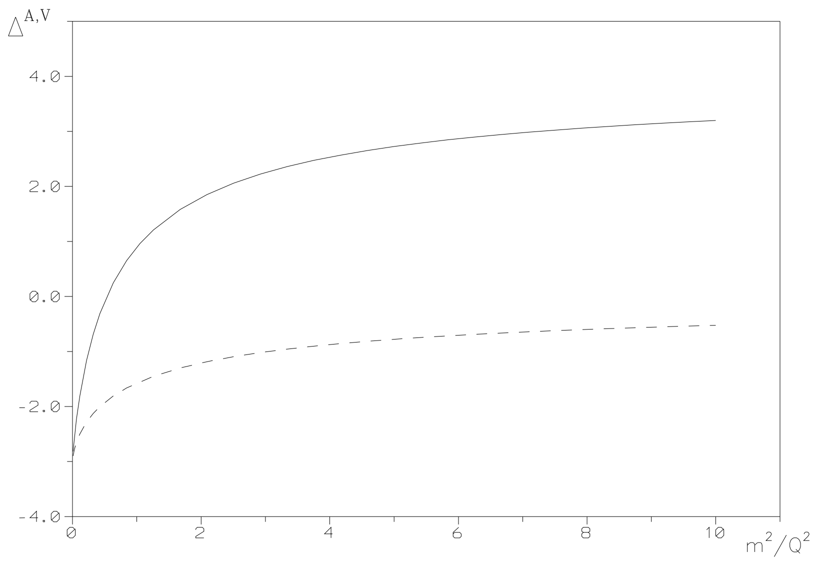

Fig. 1. Corrections to the Bjorken (solid line) and Gross–Llewellyn-Smith (dashed line) sum rules versus m^2/Q^2. Coefficient functions are written in the form $C^{(A),(V)} = 1 + (\alpha_s/4\pi)C_F\Delta^{A,V}$.